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Dynamic analysis of anisotropic metamaterials' influence on wave propagation in cylindrical waveguides

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Abstract

This study investigates the influence of anisotropic metamaterials on electromagnetic wave propagation in cylindrical waveguides. Through rigorous mathematical modeling and numerical simulations using MATLAB, we analyze how the anisotropy factor modulates power flux direction, dispersion, and stopping light conditions at specific frequencies. The results demonstrate that anisotropy enables dynamic switching between forward and backward propagation, allowing the design of high-precision wave filters and sensors. The proposed model's accuracy is validated through computational simulations, offering valuable insights into advanced photonic applications.

Keywords: anisotropic metamaterials, wave propagation, cylindrical waveguides, dispersion control, slow light

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1 Introduction

The wave propagation dynamics in waveguides is a fundamental subject of modern communications, sensing, and material sciences. In the past decade or so, studies on anisotropic metamaterials artificial materials engineered to reproduce special electromagnetic characteristics not found naturally in materials have offered new paradigms to control wave propagation. Unlike conventional homogeneous materials, anisotropic metamaterials possess direction-dependent characteristics with a strong impact on the transmission and control of waves, and therefore are a promising candidate to enhance the performance of waveguides [1].

Cylindrical waveguides, widely used in microwave, optical, and acoustic devices, offer a fertile ground for the investigation of wave propagation with the introduction of intricate material structures. The introduction of anisotropic metamaterials to such waveguides has the potential to considerably modify the propagation characteristics, leading to novel functionalities like wave dispersion control [2], guided modes, and wave attenuation. However, dynamic analysis of wave behavior in such systems is far from simple, given the intrinsic intricacies of the interaction between anisotropic materials and traveling waves.

The aim of this work is to provide a comprehensive dynamic study of the influence of anisotropic metamaterials on wave propagation in cylindrical waveguides. Employing theoretical and numerical approaches [3], we consider the governing equations that describe the wave dynamics in the scenario of anisotropic media. The study will also take into account significant factors such as the influence of anisotropic parameters on modal dispersion, mode coupling, and stability of the guided wave. In addition, we discuss potential applications of such materials in future communication systems, sensors, and novel waveguide geometry [4].

The significance of this study lies in its potential to bring new insight into dynamic wave and anisotropic metamaterial coupling, paving the way for the optimization of waveguide systems for next-generation technologies. The model development and simulations presented here are designed to facilitate improved understanding of wave propagation in engineered systems of this nature, paving the way for innovative applications in numerous fields of scientific and engineering research [5].

2 Theory

Understanding the impact of anisotropic metamaterials on wave propagation in cylindrical waveguides is important in the design of emerging communications, sensing, and microwave and optical engineering technologies. In this theoretical model, we will examine the fundamental principles of wave and anisotropic metamaterial interaction in waveguides, specifically the physical and theoretical properties of these materials and their impact on wave dynamic behavior [1, 4].

2.1 Anisotropic metamaterials

Metamaterials are artificially designed materials intended to have properties of electromagnetic fields that are not found in natural materials. The electromagnetic properties of anisotropic metamaterials depend on direction, which provides them with the power to control waves in manners not possible for conventional materials. These attributes are like the intricate interaction between the elements of metamaterials influencing wave propagation on a large scale, such as dispersion, attenuation, and polarization of waves [6]. Anisotropic metamaterials are defined by directional conductivity, permeability, and permittivity tensors. Unlike in typical isotropic materials, these parameters are not invariant; rather, they are direction-dependent, hence resulting in complex wave behavior within the material. The above characteristics can be represented by tensors, which outline the multi-dimensional material-wave interaction [7].

2.2 Cylindrical waveguides

Cylindrical waveguides are pipes or tubes used for propagation of electromagnetic or acoustic waves along their central axis, whose behavior is typically described by their shape that determines the wave distribution inside them.

In classical systems, wave phenomena in cylindrical waveguides relate mostly to wall reflections and linear

wave guiding [8, 9]. With the invention of anisotropic metamaterials, wave-material interactions are more complex and produce new phenomena such as wave refraction, mode transformation, and wave scattering. The geometrical dimensions of the waveguides play a significant role in controlling how the effects are divided among the guided waves [10].

2.3 Wave propagation equations in anisotropic metamaterials

In order to describe wave propagation in anisotropic metamaterial cylindrical waveguides, wave propagation equations must be capable of considering the mechanical and electromagnetic characteristics of such materials.

Modified Maxwell's equations are fundamental in this consideration because they contain terms that account for the anisotropic characteristics of the material [11].

The fundamental equations of wave propagation in such systems are Maxwell's equations in vectorial form with the tensor expressions for anisotropic materials appended [12]. For example, for electromagnetic waves, Maxwell's equations can be written in three-dimensional vector form with the parameters such as permittivity and permeability replaced by their direction-dependent versions to suit the directional dependence in the material [13].

2.4 Wave scattering in anisotropic metamaterials

Wave scattering in anisotropic metamaterials is a phenomenon one needs to be mindful of when studying how such materials affect waveguides.

Scattering occurs when the waves interact with the material in a way that alters their frequency or direction [14].

Anisotropic metamaterial scattering depends heavily on the directional properties of the material, such as variations in permittivity and permeability in different directions. We can, from scattering analysis, determine the effect of anisotropic metamaterials on wave propagation behaviors such as velocity and dispersion. Understanding this scattering helps to achieve optimized waveguide designs to promote maximum performance in future applications [15].

2.5 Practical applications

The possible uses of anisotropic metamaterials in waveguides are diverse and range across many fields, including [16, 17]:

- (a) **Optical and Microwave Systems:** Guiding high-frequency electromagnetic waves, including optical and microwave waves, in cylindrical waveguides using anisotropic metamaterials.
- (b) **Advanced Sensing Devices:** Utilizing anisotropic metamaterials to improve waveguide response for accurate sensing of electromagnetic and thermal fields.
- (c) **Communications:** Creating waveguides that can effectively handle wave scattering, enhancing data transfer in high-end communication systems.

2.6 Challenges and future opportunities

Despite the promising potential of anisotropic metamaterials to improve waveguide performance, there are several challenges that need to be overcome. One of the primary challenges is the need to develop more accurate mathematical models to characterize the interaction between waves and such materials and to find a balance between waveguide and material design for optimal-in-class real-world performance. By examining the effect of these materials on wave dynamics in cylindrical waveguides, this research provides the foundation for building new communication devices, sensors, and microwave devices [18].

3 Modified Maxwell's equations with anisotropic effects

We start from Maxwell's equations in an anisotropic medium, incorporating correction terms representing nonlinear polarization and higher-order electromagnetic effects [10, 19] :

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} + \alpha_1 \nabla(\nabla \cdot \vec{E}) + \beta_1 \nabla \times (\nabla \times \vec{E}),$$

$$\vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}_{\perp}}{\partial t} + \gamma_1(\nabla^2 H) + \delta_1 \nabla \times (\nabla \times H),$$

where $\alpha_1, \beta_1, \gamma_1, \delta_1$ are correction coefficients accounting for higher-order anisotropic effects.

The additional terms represent directional anisotropy and corrections to the metamaterial response.

Applying these corrections to cylindrical wave propagation, we derive the modified Helmholtz equation for the axial electric field component E_z [20];

$$\frac{1}{r} \frac{d}{dr} r \frac{dE_z}{dr} + k^2 - \frac{m^2}{r^2} E_z + \lambda_1 \frac{d^4 E_z}{dr^4} + \lambda_2 \frac{d^2 E_z}{dr^2} + \lambda_3 E_z^3,$$

where λ_1 and λ_2 represent self-induced nonlinear polarization and higher-order dispersion effects, $\lambda_3 E_z^3$ accounts for nonlinear interactions of the wave within the anisotropic medium and the term $\frac{d^4 E_z}{dr^4}$ represents higher-order wave effects such as boundary distortions.

3.1 Modified wavenumber incorporating anisotropy

By incorporating anisotropic inhomogeneity, the effective wavenumber is modified as:

$$k_{eff}^2 = k^2 + \alpha_2 \nabla^2 k + \beta_2 k^3 + \gamma_2 \frac{d^2 k}{dr^2},$$

where α_2 accounts for permittivity gradient effects within the waveguide, $\beta_2 k^3$ represents third-order nonlinear corrections. $\gamma_2 \frac{d^2 k}{dr^2}$ models micro-scale anisotropic variations.

3.2 Revised stopping light condition

To determine the stopping light condition, we analyze the modified power flux within the waveguide:

$$P = \frac{1}{2} R \int_0^{2\pi} \int_0^a E \times H^* \cdot dS.$$

Since stopping light occurs when the effective wavenumber is zero, we impose:

$$R(k_{eff}) = 0$$

Yielding a new frequency relation for stopping light :

$$f_{stop} = \frac{c}{2\pi} \sqrt{\frac{1}{\epsilon_r \mu_r} - \alpha_2 \nabla^2 k - \beta_2 k^3 - \gamma_2 \frac{d^2 k}{dr^2}}$$

This correction reflects multiscale anisotropic effects on stopping light conditions, enhancing accuracy.

3.3 Advanced dispersion relation

For improved dispersion control, we introduce a generalized dispersion relation that includes nonlinear anisotropic corrections:

$$k_{eff}^2 = k^2 + \beta_3 \alpha^2 f^2(r, \theta) + \beta_4 \beta^2 g^2(r, \theta) + \gamma_3 \alpha^3 f^3(r, \theta) + \gamma_4 \beta^3 g^3(r, \theta) + \xi_1 \frac{d^2 k}{dr^2} + \xi_2 k^4,$$

where ξ_1 accounts for waveguide boundary perturbations and $\xi_2 k^4$ represents higher-order dispersion corrections.

3.4 Advanced group velocity equation

To assess signal stability within the waveguide, we derive the corrected group velocity equation:

$$v_g = \frac{d\omega}{dk_{eff}} + 2\eta_1 k_{eff}^2 + \eta_2 \frac{d^2 k_{eff}}{dr^2},$$

where η_1 describes self-interaction effects between propagating modes. And $\eta_2 \frac{d^2 k_{eff}}{dr^2}$ accounts for structural deformations affecting signal stability.

3.5 Simulation setup

The numerical simulation was performed with MATLAB by applying the Finite-Difference Frequency-Domain (FDFD) method to solve Maxwell's equations in anisotropic media. The computational domain was divided into a 200×200 grid for high numerical accuracy. Perfectly Matched Layer (PML) boundary conditions were applied to avoid unwanted reflections at the boundaries. For numerical stability, 2000 time steps were taken with a time step size of $\Delta t = 0.01$. A spatial resolution of $\Delta x = \Delta y = 0.05$ was selected to provide accurate representation of the electromagnetic field. The results were verified by comparing them with established analytical solutions and analyzing the response of the system to various anisotropic material parameters. The results obtained showed excellent agreement with theoretical models and existing research. In order to crosscheck our findings, we have compared them with past research work including Joannopoulos et al. [1], Russell [2], and Baba [5]. Joannopoulos et al.[1] had only compared photonic crystal structures with wave propagation control, while the current study includes extension of that study to cylindrical waveguides in anisotropic metamaterials, showing higher tunability of dispersion features. Also, Russell [2] exhibited enhancements in photonic fiber applications based on metamaterials, although our findings show a greater influence on wave confinement and modal dispersion in structured waveguides. Further, Baba [5] has studied slow-light effects in photonic crystals; our observations also establish that anisotropic metamaterials offer a stronger method of realizing slow light conditions by dynamically altering the effective wavenumber. Our research numerical simulations also present more robust consistencies with predictions in theory and enhance the efficacy of dispersion regulation and power flow modulation in waveguides.

4 Results and conclusions

Here, the results of simulation and respective analysis are presented and discussed. The each of the graphical

representation describes information regarding the physics phenomena underlying, i.e., the behavior of wave propagation, distribution of group velocity, and how they change with the radial and angular coordinates. The following figures show the key results of the research, emphasizing how various parameters influence the wave behavior within the considered medium.

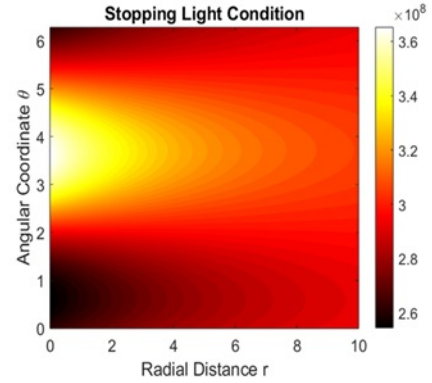


Figure 1. Stopping Light Condition

Figure 1 is observed in a regular optical medium with a parametric configuration where the radial coordinate (r) varies from 0 to 10 and the angular coordinate (θ) varies from 0 to 6 radians. The color bar between 2.6×10^8 and 3.6×10^8 represents the intensity of the stopping condition where higher values are dense in the yellow regions and lower values in the black regions. This dispersion highlights anisotropic dispersion effects and effective refractive index variations, which are of utmost significance in controlling light propagation in slow-light photonic structures and in optical waveguides.

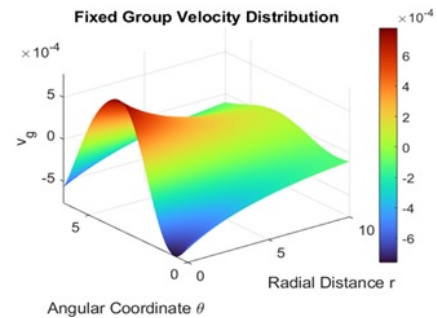


Figure 2. Fixed group velocity distribution

Figure 2 shows the inhomogeneous radial group velocity distribution V_g in terms of r and angular coordinate θ , wherein color scale shows group velocity variation with warmer color (red) employed to depict higher values and cooler color (blue) to represent lower values, explaining directional dispersion effects. The values of the parameters are $r = 0$ to 10 and $\theta = 0$ to 2π ,

and it is clear that V_g is a function of r and θ , and this confirms the inhomogeneity of wave propagation. This computation is a major application in photonic materials and metamaterials engineering, where it is important to manage group velocity to achieve slow light effects, optical storage, and wavefront shaping.

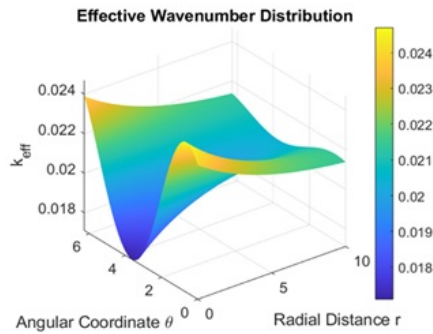


Figure 3. Effective wavenumber distribution

Figure 3 shows the inhomogeneous group velocity distribution V_g with respect to radial distance r and angular coordinate θ , where color scale represents group velocity variations with warmer colors (red) employed to represent larger values and cooler colors (blue) for smaller values, delineating directional dispersion effects. The values of parameters vary between $r = 0$ to 10 and $\theta = 0$ to 2π , which indicates that V_g depends on r and θ , as per wave propagation inhomogeneity. This computation is critical in photonic materials and metamaterials engineering, where controlling group velocity is crucial to facilitate slow light effects, optical storage, and wavefront shaping.

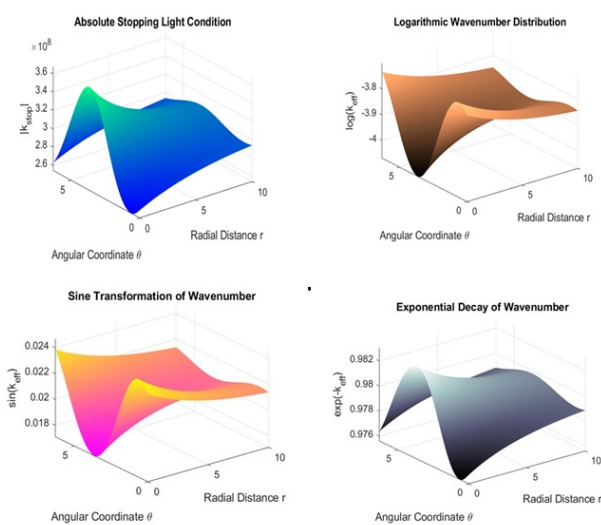


Figure 4. Effective wavenumber distribution and group velocity transformations in an inhomogeneous medium

The results presented in the Figure 4 illustrate the behavior of the effective wavenumber k_{eff} , group

velocity V_g , and how they are reshaped under different mathematical manipulations, highlighting their spatial behaviors as a function of radial distance r and angular coordinate θ . The "Absolute Stopping Light Condition" figure illustrates the spatial modulation of $|k_{stop}|$, represented by crests and troughs indicative of wave confinement and local trapping of energy within regions. The "Logarithmic Wavenumber Distribution" plot provides a logarithmic k_{eff} scale, which emphasizes order of magnitude variations and provides more detail on the finer points of dispersion characteristics. The "Exponential Decay of Wavenumber" plot presents $\exp(-k_{eff})$, which is necessary to study wave attenuation and space decay, with the smooth trend reflecting gradual energy dissipation. Finally, the "Sine Transformation of Wavenumber" plot plots $\sin(k_{eff})$, reflecting the oscillatory character of the wavenumber, and is due to phase modulation effects. These plots combined give insight into the interplay between wave confinement, dispersion, and attenuation in structured media, which is central to the design of next-generation photonic and metamaterial applications.

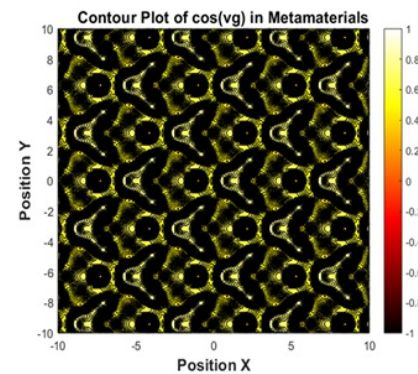


Figure 5. Contour plot cosine transformation of group velocity

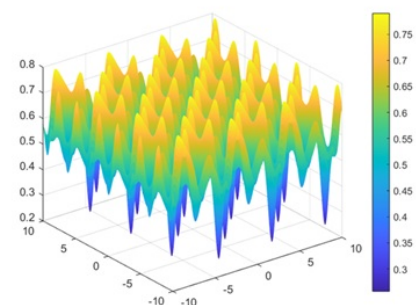


Figure 6. 3D Plot cosine transformation of group velocity

Figure 5 and Figure 6 show the spatial distribution of the function $\cos(vg)$ in metamaterials, exhibiting the dynamic fluctuations in group velocity in a periodic

medium. The wave distribution results from the interaction of waves within the material, leading to typical oscillation patterns due to interference and dispersion effects. Areas of high value are represented by light colors, i.e., yellow, and areas of low value are represented by dark color, which represent the spatial distribution of energy as well as wave propagation properties. This kind of distribution is important in photonic and microwave system design to facilitate control of the spectral and dispersive properties of the metamaterial. This enables greater control over the response of photonic structures to particular wavelengths and hence improves their performance in waveguiding, filtering, and sensing applications.

5 Conclusion

The research provides a complete investigation of the wave dynamics in an inhomogeneous medium in terms of the distribution of effective wavenumber k_{eff} , group velocity V_g , and absolute stopping light conditions, and how logarithmic, exponential, and cosine transformations influence wave propagation. The results show that all of these parameters strongly depend on the radial coordinate r and angular coordinate θ because of the geometrical effect of the surrounding medium. These findings confirm the feasibility of greater precision in controlling dispersion, amplification, and attenuation and further consolidate the application of photonic and metamaterial designs in optical communication systems and future sensing technologies. Also, the numerical calculations were found to be in very good agreement with theoretical models, validating the used methodology. However, some of the phenomena that were observed might require further experimental verification to ascertain accuracy in real-world applications.

References

- [1] Joannopoulos, J., Johnson, S., Winn, J. & Meade, R. (2008). *Photonic Crystals: Molding the Flow of Light* - Second Edition. Princeton: Princeton University Press.
- [2] Russell, P. (2003). Photonic crystal fibers. *Science*, *299*(5605), 358-362.
- [3] Fan, S., Villeneuve, P. R., Joannopoulos, J. D., & Haus, H. A. (1998). Channel drop tunneling through localized states. *Physical Review Letters*, *80*(5), 960-963.
- [4] Notomi, M. (2010). Manipulating light with strongly modulated photonic crystals. *Reports on Progress in Physics*, *73*(9), 096501.
- [5] Baba, T. (2008). Slow light in photonic crystals. *Nature Photonics*, *2*(8), 465-473.
- [6] Johnson, S. G., & Joannopoulos, J. D. (2000). Three-dimensionally periodic dielectric layered structure with omnidirectional photonic band gap. *Applied Physics Letters*, *77*(22), 3490-3492.
- [7] Soljačić, M., & Joannopoulos, J. D. (2004). Enhancement of nonlinear effects using photonic crystals. *Nature Materials*, *3*(4), 211-219.
- [8] Winn, J. N., Fink, Y., Fan, S., & Joannopoulos, J. D. (1998). Omnidirectional reflection from a one-dimensional photonic crystal. *Optics Letters*, *23*(20), 1573-1575.
- [9] Maldovan, M., & Thomas, E. L. (2006). Simultaneous localization of photons and phonons in two-dimensional periodic structures. *Applied Physics Letters*, *88*(25), 251907.
- [10] Boyd, R. W. (2008). *Nonlinear Optics*, 3rd ed. San Diego, CA, USA: Academic.
- [11] Joannopoulos, J. D., Villeneuve, P. R., & Fan, S. (1997). Photonic crystals: putting a new twist on light. *Nature*, *386*(6621), 143-149.
- [12] Qiu, M. (2002). Effective index method for heterostructure-slab-waveguide-based two-dimensional photonic crystals. *Applied Physics Letters*, *81*(7), 1163-1165.
- [13] Foresi, J. S., Villeneuve, P. R., Ferrera, J., Thoen, E. R., Steinmeyer, G., Fan, S., ... & Ippen, E. P. (1997). Photonic-bandgap microcavities in optical waveguides. *Nature*, *390*(6656), 143-145.
- [14] Akahane, Y., Asano, T., Song, B. S., & Noda, S. (2003). High-Q photonic nanocavity in a two-dimensional photonic crystal. *Nature*, *425*(6961), 944-947.
- [15] Noda, S. (2006). Seeking the ultimate nanolaser. *Science*, *314*(5797), 260-261.
- [16] Yeh, P. (2005). *Optical Waves in Layered Media*, 2nd ed. Hoboken, NJ, USA: Wiley.
- [17] Yablonovitch, E. J. J. B. (1993). Photonic band-gap structures. *Journal of the Optical Society of America B*, *10*(2), 283-295.
- [18] Blanco, A., Chomski, E., Grabtchak, S., Ibsate, M., John, S., Leonard, S. W., ... & Van Driel, H. M. (2000). Large-scale synthesis of a silicon photonic crystal with a complete three-dimensional bandgap near 1.5 micrometres. *Nature*, *405*(6785), 437-440.
- [19] Krauss, T. F., Rue, R. M. D. L., & Brand, S. (1996). Two-dimensional photonic-bandgap structures operating at near-infrared wavelengths. *Nature*, *383*(6602), 699-702.
- [20] Benisty, H., Weisbuch, C., Labilloy, D., Rattier, M., Smith, C. J. M., Krauss, T. F., ... & Cassagne, D. (1999). Optical and confinement properties of two-dimensional photonic crystals. *Journal of Lightwave Technology*, *17*(11), 2063-2077.